

# Effective Quark Models in QCD at low and intermediate energies\*

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## Abstract

The effective quark models are employed to describe the hadronization of QCD in the quark sector. They reveal a different structure depending on how the spontaneous chiral symmetry breaking (CSB) is implemented. When the generation of light pseudoscalar mesons is manifestly incorporated one deals with an extension of the chiral quark model (CQM) with the non-linear realization of chiral symmetry. If a model is built at the CSB scale by means of perturbation theory it generalizes the Nambu-Jona-Lasinio (NJL) one with chiral symmetry broken due to attractive 4-fermion forces in the scalar channel. The matching to high-energy QCD is realized at CSB scale by means of Chiral Sum Rules. Two types of models are compared in their fitting of meson physics. In particular, if the lowest scalar meson is sufficiently heavy approaching the mass of heavy  $\pi'(1300)$  then QCD favours an effective theory which is dominated by the simplest CQM. On the contrary, a light scalar quarkonium ( $m_\sigma \simeq 500 MeV$ ) supports the NJL mechanism.

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# 1. Introduction

The notion of quark model appears in the literature for many different truncations of QCD: when one makes a request to SLAC-SPIRES service to FIND TITLE QUARK MODEL the result exceeds 3700 documents. If one assumes that for each version there are around 100 papers with its application, one ends up with about 30-40 different quark models. Among them there are many which simulate a part of gluon interaction in the form of a potential or specific nonlocal forces and thereby do not give a controllable truncation of QCD. We leave aside the bulk of those quark models and we focus only on two type of models which open the way of a systematic construction of the low-energy effective action of QCD in the quark sector. In the models we are considering the gluon interaction is hidden in the effective coupling constants. In their simplest form they have been used for a long time to reproduce main features of QCD hadronization [1-4].

The first one is the Chiral Quark Model (CQM) [1] with quark fields  $q_i^\alpha, \bar{q}_i^\alpha$ ; carrying a color,  $i = 1, \dots, N_c$ , and a flavour,  $\alpha = 1, 2, \dots, N_F$  and interacting with the colorless chiral field  $U(x)$ ,

$$\mathcal{L}_{CQM} = \bar{q} \left( i \not{\partial} - M_0 (U P_L + U^\dagger P_R) \right) q + \frac{1}{4} f_0^2 \text{tr} [\partial_\mu U \partial^\mu U^\dagger], \quad (1)$$

where  $\not{\partial} \equiv \gamma^\mu \partial_\mu$  and the projectors  $P_{L(R)} = (1 \pm \gamma_5)/2$  are used.  $M_0$  is a chirally invariant constituent mass, which has a non-zero value once chiral symmetry is spontaneously broken. For two light flavors  $U(x) = \exp(i\pi(x)/F_0)$  is a  $SU(2)$  matrix with generators  $\pi \equiv \pi^a T^a$ ,  $a = 1, 2, 3$ ; corresponding to the massless Goldstone bosons – pions.

The constant  $F_0$  is the pion decay constant, whereas  $f_0$  represents a bare pion decay constant [2], which contains residual gluon contributions not accounted for by the radiative quark effects.

The second approach is provided by the Nambu-Jona-Lasinio (NJL) model [3]. It includes a chirally invariant four-fermion interaction. For massless quarks it reads

$$\mathcal{L}_{NJL} = \bar{q} i \not{\partial} q + \frac{g_0}{4N_c \Lambda^2} \left[ (\bar{q} q)^2 - (\bar{q} \gamma_5 \tau^a q)^2 \right]. \quad (2)$$

In this model dynamical breaking of chiral symmetry occurs for strong enough coupling  $g_0$ . This leads to the creation of a massless pion state, to the appearance of a dynamical quark mass  $\Sigma_0$  (similar to  $M_0$ ), and to the generation of a scalar meson with the mass  $m_\sigma \simeq 2\Sigma_0$ , the  $\sigma$ -particle.

Both models provide the similar set of low energy structural constants describing interactions between pions and therefore may serve for interpolating the true low-energy effective action of QCD.

## 2. Extension of Quark Models

For intermediate energies the above models need to be extended [5-8] in order to get a better agreement with meson phenomenology and to satisfy the requirements of QCD.

First, one may build the most general action describing strong interactions once the spontaneous CSB has taken place at a scale  $\Lambda$ . This effective action should contain the lightest degrees of freedom appearing below  $\Lambda$ , namely the (pseudo) Goldstone bosons – pions (for two flavours), assembled in the chiral field  $U$ . One has to include all local operators -gauge and chiral invariant, which can be composed of the chiral field  $U$  and quarks. Their list contain already about 50 operators (see [7]) of  $\dim \leq 6$  but only a few of them contribute to the leading order in the large-cutoff expansion.

We embed the external sources into the QCD quark lagrangian in order to compute the correlators of corresponding quark currents,

$$\hat{D} \equiv i\gamma_\mu(\partial_\mu + \bar{V}_\mu + \gamma_5 \bar{A}_\mu) + i(\bar{S} + i\gamma_5 \bar{P}), \quad (3)$$

where  $\langle S \rangle = m_q$ , the matrix of current quark masses.

To describe the Extended CQM (ECQM) action it is convenient to introduce the ‘constituent’ quark fields  $Q_L \equiv \xi q_L$ ,  $Q_R \equiv \xi^\dagger q_R$ ,  $\xi^2 \equiv U$ , which transform nonlinearly under  $SU_L(2) \otimes SU_R(2)$  but identically for left and right quark components.

Changing to this basis is given by the following replacements in the external vector, axial, scalar and pseudoscalar sources

$$\begin{aligned} \bar{V}_\mu &\rightarrow v_\mu = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \partial_\mu \xi \xi^\dagger + \xi^\dagger \bar{V}_\mu \xi + \xi \bar{V}_\mu \xi^\dagger - \xi^\dagger \bar{A}_\mu \xi + \xi \bar{A}_\mu \xi^\dagger \right), \\ \bar{A}_\mu &\rightarrow a_\mu = \frac{1}{2} \left( -\xi^\dagger \partial_\mu \xi - \partial_\mu \xi \xi^\dagger - \xi^\dagger \bar{V}_\mu \xi + \xi \bar{V}_\mu \xi^\dagger + \xi^\dagger \bar{A}_\mu \xi + \xi \bar{A}_\mu \xi^\dagger \right), \\ \bar{\mathcal{M}} &\equiv (\bar{S} + i\bar{P}) \rightarrow \mathcal{M} = \xi^\dagger \bar{\mathcal{M}} \xi^\dagger. \end{aligned} \quad (4)$$

In these variables the relevant part of ECQM action can be represented as,

$$\mathcal{L}_{ECQM} = \mathcal{L}_{ch} + \mathcal{L}_{\mathcal{M}}, \quad (5)$$

where  $\mathcal{L}_{ch}$  accumulates the interaction of chiral fields and quarks in the chiral limit in the presence of vector and axial-vector external fields,

$$\begin{aligned} \mathcal{L}_{ch} = & i\bar{Q} (\not{D} + M_0) Q - \frac{f_0^2}{4} \text{tr}(a_\mu^2) + \frac{G_{S0}}{4N_c\Lambda^2} (\bar{Q}_L Q_R + \bar{Q}_R Q_L)^2 \\ & - \frac{G_{P1}}{4N_c\Lambda^2} (-\bar{Q}_L \vec{\tau} Q_R + \bar{Q}_R \vec{\tau} Q_L)^2 \\ & - \frac{G_{V1}}{4N_c\Lambda^2} \bar{Q} \vec{\tau} \gamma_\mu Q \bar{Q} \vec{\tau} \gamma_\mu Q - \frac{G_{A1}}{4N_c\Lambda^2} \bar{Q} \vec{\tau} \gamma_5 \gamma_\mu Q \bar{Q} \vec{\tau} \gamma_5 \gamma_\mu Q \\ & + c_{10} \text{tr}[U \bar{L}_{\mu\nu} U^\dagger \bar{R}_{\mu\nu}], \end{aligned} \quad (6)$$

where

$$Q \equiv Q_L + Q_R, \quad \not{D} \equiv \not{\partial} + \not{\partial} - \gamma_5 \tilde{g}_A \not{A}, \quad (7)$$

with the ‘bare’ axial coupling  $\tilde{g}_A \equiv 1 - \delta g_A$  and the ‘bare’ chiral coupling  $c_{10}$ .

$\mathcal{L}_{\mathcal{M}}$  extends the description for external scalar and pseudoscalar fields and, in particular, for massive quarks,

$$\begin{aligned} \mathcal{L}_{\mathcal{M}} = & i\left(\frac{1}{2} + \epsilon\right) \left(\bar{Q}_R \mathcal{M} Q_L + \bar{Q}_L \mathcal{M}^\dagger Q_R\right) + i\left(\frac{1}{2} - \epsilon\right) \left(\bar{Q}_R \mathcal{M}^\dagger Q_L + \bar{Q}_L \mathcal{M} Q_R\right) \\ & + \text{tr} \left[ c_0 (\mathcal{M} + \mathcal{M}^\dagger) + c_5 (\mathcal{M} + \mathcal{M}^\dagger) a_\mu^2 + c_8 \left( \mathcal{M}^2 + (\mathcal{M}^\dagger)^2 \right) \right], \end{aligned} \quad (8)$$

where the chiral couplings  $c_0, c_5, c_8$  (as well as  $c_{10}$ ) are ‘bare’, different from those introduced in [9]. All ‘bare’ coupling constants are renormalized by radiative quark contributions and their physical values are controlled by the CSR rules (see below).

Thus the effective action suitable for derivation of two-point correlators contains 13 parameters to be determined by matching to QCD:  $M_0, \Lambda(\text{cutoff})$ , bare chiral constants  $f_0, c_0, c_5, c_8, c_{10}$ , the axial pion-quark coupling  $\tilde{g}_A$ , the mass asymmetry  $\epsilon$ , the four-fermion coupling constants  $G_{S0} \neq G_{P1}, G_{V1} \neq G_{A1}$ . In this approach the pion plays a very crucial role, quite different from other hadronic resonances.

On the other hand one could construct the QCD effective action employing the perturbative expansion in the QCD coupling constant  $\alpha_s$  for the high-energy gluons and quarks [10]. The effective action for low-energy quarks is further prepared by means of the derivative (soft-momentum) expansion, generating an infinite series of quasilocal higher-dimensional operators. Then for sufficiently strong couplings, the new operators may promote the generation of additional scalar and pseudoscalar states. These models give an extension of the linear  $\sigma$  model provided by the NJL model, with the pion being a broken symmetry partner of the lightest scalar meson, and with excited pions and scalar particles, as well as with vector and axial-vector mesons, coming in pairs. In particular, when scalar, pseudoscalar, vector and axial-vector color-singlet channels are examined and dynamical quark masses are supposed to be sufficiently smaller than the CSB cutoff one may derive the minimal two-channel lagrangian in the separable form:

$$\begin{aligned} \mathcal{L}_{ENJL} = & i\bar{q}\hat{D}q + \frac{1}{4N_c\Lambda^2} \sum_{i,k=0,1} (a_{ik} [\bar{q}f_i q \bar{q}f_k q - \bar{q}\gamma_5\tau^a f_i q \bar{q}\gamma_5\tau^a f_k q] \\ & - b_{ik} [\bar{q}\gamma_\mu\tau^a f_i q \bar{q}\gamma_\mu\tau^a f_k q - \bar{q}\gamma_5\gamma_\mu\tau^a f_i q \bar{q}\gamma_5\gamma_\mu\tau^a f_k q]), \end{aligned} \quad (9)$$

where

$$f_1(\hat{s}) = 2 - 3\hat{s}; \quad f_2(\hat{s}) = -\sqrt{3}\hat{s}; \quad \hat{s} \equiv -\frac{\partial^2}{\Lambda^2}. \quad (10)$$

With the CSB momentum cutoff  $\Lambda$  one can specify the critical values of coupling constants above which the dynamical CSB occurs,  $a_{kl} \simeq 8\pi^2\delta_{kl}$ . Eventually, two multiplets of scalar, pseudoscalar, vector and axial-vector mesons are generated in the vicinity of these critical constants.

### 3. Bosonization and technology

First we consider the ECQM. To find the characteristics of composite boson states in colorless quark channels we incorporate auxiliary fields  $\Phi$  in the scalar and pseudoscalar channels,  $\Sigma, \Pi^a$ , and in the vector and axial-vector channel,  $W_\mu^{(\pm)a}$ , and replace the four-fermion operators by

$$\begin{aligned}\mathcal{L}_{4-quark} &= \bar{Q}\Gamma\Phi Q + N_c\Lambda^2 \frac{\Phi^2}{G_C}; \\ \Gamma &= 1; i\gamma_5\tau^a; i\gamma_\mu\tau^a; i\gamma_5\gamma_\mu\tau^a; \quad C = S0, P1, V1, A1;\end{aligned}\tag{11}$$

with an integration over new variables.

The actual value of constituent mass  $\langle \Sigma \rangle = \Sigma_0$  in the ECQM is controlled [7] by the mass-gap equation (the minimum of the scalar effective potential)

$$\frac{\Lambda^2}{G_{S0}}(\Sigma_0 - M_0) = -\frac{\Sigma_0^3}{4\pi^2} \ln \frac{\Lambda^2}{\Sigma_0^2} \equiv \Sigma_0^3 I_0.\tag{12}$$

Therefrom it is evident that the natural scale for the four-fermion interaction is given by  $\Sigma_0$  rather than by  $\Lambda$  and it is useful to redefine the related coupling constants:  $\bar{G}_C = G_C I_0 \frac{\Sigma_0^2}{\Lambda^2}$ , characterizing the weak coupling regime when  $\bar{G}_C \ll 1$ .

The evaluation of physical characteristics in the ENJL model is similarly based on the bosonization in terms of auxiliary scalar and pseudoscalar fields:

$$\begin{aligned}\mathcal{L}_{4-quark} &= \sum_{k=1}^2 i\bar{q} \left( \sigma_k + i\gamma_5 \pi_k^+ \gamma_\mu \rho_{k,\mu} + \gamma_5 \gamma_\mu a_{k,\mu} \right) f_k(\hat{s}) q \\ &+ N_c \Lambda^2 \sum_{k,l=1}^2 \left( \sigma_k a_{kl}^{-1} \sigma_l + \pi_k^a a_{kl}^{-1} \pi_l^a \rho_{l,\mu}^a b_{kl}^{-1} \rho_{l,\mu}^a + a_{l,\mu}^a b_{kl}^{-1} a_{l,\mu}^a \right).\end{aligned}\tag{13}$$

Let us parametrize the matrix of coupling constants in a close vicinity of tricritical point:

$$8\pi^2 a_{kl}^{-1} = \delta_{kl} - \frac{\Delta_{kl}}{\Lambda^2}, \quad |\Delta_{kl}| \ll \Lambda^2.\tag{14}$$

The last inequality is equivalent to require the dynamical mass to be essentially less than the cutoff. After integrating out the quark fields one comes to the bosonic effective action  $\mathcal{W}(\sigma_k, \pi_k^a, \rho_{l,\mu}, a_{l,\mu})$ . The conditions on extremum of the effective potential, are given by the mass-gap equations [5].

In both models the auxiliary fields obtain kinetic terms from the quark loops and propagate, i.e. interpolate scalar pseudoscalar, vector and axial-vector resonance states.

The approximation is consistent for a finite number of resonances if one retains only the part of the soft-momentum expansion of quark loop together with a finite number of vertices in the Quark Model effective action.

This approach respects the confinement requirement, because one coherently neglects both the threshold part of quark loop and (the infinite number of) heavier resonance poles. This is supported by the large- $N_c$  approximation which associates all momentum dependence in the bosonized action solely with meson resonances.

An additional simplification is achieved with the help of the leading-log approximation when one retains only the leading logarithm of the cutoff  $\Lambda$  which is a RG universal part of quark loops. In this approximation we lose somewhat in precision but gain the predictability and model independence.

## 4. Matching to QCD

Let us exploit the constraints based on chiral symmetry restoration at QCD at high energies. We focus on two-point correlators of colorless quark currents

$$\Pi_C(p^2) = \int d^4x \exp(ipx) \langle T (\bar{q}\Gamma q(x) \bar{q}\Gamma q(0)) \rangle, \quad (15)$$

where  $C = S, P, V, A$  and respectively  $\Gamma = 1, i\gamma_5, \gamma_\mu, \gamma_5\gamma_\mu$ . In the chiral limit the scalar correlator and the pseudoscalar one, as well as the vector correlator and the axial-vector one, coincide at all orders in perturbation theory and also at leading order in the non-perturbative O.P.E.[11]. As the differences  $\Pi_S - \Pi_P$  and  $\Pi_V - \Pi_A$  decrease rapidly with growing momenta, one can expect that the lowest lying resonances included into a Quark Model essentially saturate the constraints from CSB restoration.

For the ECQM and the ENJLM, in the scalar channel, one obtains the following sum rules [7, 12, 13],

$$c_8 + \frac{N_c \Sigma_0^2 I_0}{8\bar{G}_S} - \frac{4\epsilon^2 N_c \Sigma_0^2 I_0}{8\bar{G}_P} = 0, \quad (16)$$

$$\sum Z_\sigma = Z_\pi + \sum Z_\Pi, \quad Z_\pi = \frac{4 \langle \bar{q}q \rangle^2}{F_0^2}, \quad (17)$$

$$\sum Z_\sigma m_\sigma^2 - \sum Z_\Pi m_\Pi^2 \simeq 24\pi\alpha_s \langle \bar{q}q \rangle^2, \quad (18)$$

where the first one is applicable only to the ECQM. It eliminates the contact term and fixes the bare constant  $c_8$ .  $Z_\sigma, Z_\pi, Z_\Pi$  stand for the residues in resonance pole contributions in the scalar and pseudoscalar correlators. In the minimal ECQM one reveals only one scalar meson with  $m_\sigma, Z_\sigma$  and in the minimal ENJLM one has two scalar mesons  $\sigma$  and  $\sigma'$ , with  $m_\sigma, m_{\sigma'}, Z_\sigma, Z_{\sigma'}$ . In both models there is a heavy pseudoscalar meson  $\Pi \equiv \pi'$ . The last relation is essentially saturated by heavy pion and heavy scalar parameters.

In the vector channel one derives the relations [12, 14]:

$$c_{10} = 0, \quad (19)$$

$$\sum f_V^2 m_V^2 = \sum f_A^2 m_A^2 + F_0^2, \quad (20)$$

$$\sum f_V^2 m_V^4 = \sum f_A^2 m_A^4, \quad (21)$$

$$\sum f_V^2 m_V^6 - \sum f_A^2 m_A^6 \simeq -8\pi\alpha_s \langle \bar{q}q \rangle^2, \quad (22)$$

where the first one is applicable only to the ECQM to fix the ‘bare’ constant  $c_{10}$ .

## 5. Fitting and comparison of ECQM and ENJLM

Let us specify the input parameters. We take  $F_0 = 90$  MeV,  $m_\pi^2 = 140$  MeV. We adopt [9]  $\hat{m}_q(1 \text{ GeV}) \simeq 6$  MeV,  $\langle \bar{q}q \rangle \simeq -(235 \text{ MeV})^3$ , and use the phenomenological value for the heavy pion mass  $m'_\pi \simeq 1300$  MeV [15]. We also take the vector and axial-vector meson masses,  $m_\rho = 770$  MeV and  $m_{a_1} \simeq 1.2$  MeV, as known parameters.

In the ECQM the fit of the scalar CSR rules (16)–(18) shows [7, 12] that they can be satisfied in the weak coupling regime of QCD as the 4-quark condensate correction is nearly irrelevant, being amount to 2% of main contribution from the heavy pion. We perform an optimal fit applying in the vector channel only CSR (19) and (20). For  $m_\sigma \simeq 1$  GeV one finds the chiral constant  $L_8 \simeq 0.8 \times 10^{-3}$ . For  $g_A = 0.55$  one obtains the chiral constant  $L_5 = 1.2 \times 10^{-3}$  ( $L_5, L_8$  to be compared with [9]) and  $\Sigma_0 \simeq 200$  MeV. Thus a rather heavy scalar meson is provided by ECQM.

Therefrom one derives that  $\bar{G}_V \simeq 0.25$ ,  $\bar{G}_A \simeq 0.2$ ,  $\bar{G}_S \simeq 0.11$ ,  $\bar{G}_P \simeq 0.13$ ,  $\tilde{g}_A \simeq 0.66$ , the bare pion coupling  $f_0 \simeq 62$  MeV and either  $\epsilon \simeq 0.05$  or  $\epsilon \simeq -0.51$ . We see that the four fermion coupling constants  $\bar{G}_S$  and  $\bar{G}_P$  as well as  $\bar{G}_V$  and  $\bar{G}_A$  are slightly different and their values are  $\ll 1$ , signifying the weak coupling regime.

However for the values  $m_\rho = 770$  MeV,  $m_{a_1} \simeq 1.2$  MeV and  $g_A = 0.55$  the two vector sum rules (21),(22) are not satisfied. They can be saturated only with additional vector multiplets.

In turn, in the ENJL one finds that the leading asymptotics (17) represents the generalized  $\sigma$ -model relation and is automatically fulfilled.

As to the second constraint the possibility to satisfy it depends on the value of the QCD coupling constants  $\alpha_s$ . Eventually it can be written [16] in terms of v.e.v.’s of two scalar fields,  $\sigma_1, \sigma_2$ :

$$m_{\sigma'}^2 - m_{\pi'}^2 \simeq 2\sigma_1^2 + \frac{4\sqrt{3}}{3}\sigma_1\sigma_2 + 6\sigma_2^2 \simeq \frac{6N_c\alpha_s}{8\pi}(\sigma_1 - \sqrt{3}\sigma_2)^2. \quad (23)$$

As the left part is always positive there exists a lower bound for  $\alpha_s \geq \frac{8\pi}{9N_c}$  providing solutions of the constraint. The lowest value of  $\alpha_s \simeq 0.9$  is given by  $\sigma_1 = -\sqrt{3}\sigma_2$  and for these v.e.v.’s one obtains the following splitting between the  $\sigma'$ - and  $\pi'$ -meson masses:

$$m_{\sigma'}^2 - m_{\pi'}^2 \simeq \frac{8}{3}\sigma_1^2 = \frac{1}{6}m_\sigma^2; \quad (24)$$

*i.e.* for  $m_\sigma = 500 \div 600 \text{ MeV}$  these masses practically coincide,  $m_{\sigma'} \simeq m_{\pi'} = 1300 \text{ MeV}$  and such a  $\sigma'$ -meson may be identified [15] with  $f_0(1300)$ . However the above value of  $\alpha_s$  lies in the region of rather strong coupling where next-to-leading corrections to the anomalous dimension of four-quark operator are not negligible,

$\sim \frac{\alpha_s}{\pi} \sim 0.3$ , and should be systematically taken into account to obtain a reasonable precision.

In the vector channel the mass spectrum exhibits the fast restoration of chiral symmetry. Namely with the above inputs for  $m_\rho$  and  $m_{a_1}$  one obtains the splitting,

$$m_{a'_1}^2 - m_{\rho'}^2 \simeq \frac{3}{2}(m_{\sigma'}^2 - m_{\pi'}^2). \quad (25)$$

Therefrom one predicts the mass of the excited  $a_1$  meson to be  $m_{a'_1} \simeq 1500$  MeV, which has not been yet discovered.

Thus we have shown that both the ECQM and the ENJLM truncating low-energy QCD effective action can serve to describe the physics of heavy meson resonances. The matching to nonperturbative QCD based on the chiral symmetry restoration at high energies guarantees the predictability of both models.

In the ECQM an optimal fit supports the existence of rather heavy scalar quarkonium with the mass of order 1 GeV. Meanwhile the ENJLM inevitably contains a rather light scalar meson with the mass 500 – 600 MeV which is however not completely excluded by the particle phenomenology [15], though this light meson is most probably a 2-pion or 4-quark bound state [17].

The fast convergence in the ENJLM of mass spectra and other characteristics of heavy parity doublers entail their decoupling from the low-energy pion physics. For instance, let us calculate the dim-4 chiral coupling constant [18],

$$L_8 = \frac{F_\pi^4}{64 \langle \bar{q}q \rangle^2} \left( \frac{Z_\sigma}{m_\sigma^2} + \frac{Z_{\sigma'}}{m_{\sigma'}^2} - \frac{Z_{\pi'}}{m_{\pi'}^2} \right) \simeq \frac{F_\pi^2}{16m_\sigma^2}, \quad (26)$$

when the CSR (17), (18) are imposed. The deviation due to  $\sigma'$ - and  $\pi'$ -mesons [16] is less than 2%. Thus this constant is essentially determined in the ENJLM by the lightest scalar meson. Its phenomenological value,  $L_8 = (0.9 \pm 0.4) \times 10^{-3}$  from [9] accepts  $m_\sigma \simeq 600$  MeV.

On the contrary, the same chiral constant in the ECQM contains a substantial contribution (up to 50%) from the heavy pion  $\pi'$ :

$$L_8 \simeq \frac{F_0^2}{16} \left( \frac{1}{m_\sigma^2} + \frac{1}{m_{\pi'}^2} \right), \quad (27)$$

with a good precision [7].

Thus the two approaches encode quite a different dynamics of heavy resonances responsible for the formation of structural chiral constants.

Some disadvantage of the ENJLM as well as of the original NJL model is that they assume large, critical values of four-quark coupling constants which is difficult



to justify with perturbative calculations in QCD. Respectively the CSR matching has to be performed at a scale where the QCD coupling constant is rather large and the perturbation theory is unreliable. The ECQM seem to be free of these shortcomings. However the final choice between them may be done by the fit of a larger variety of meson characteristics which is in progress.

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